Rikitake dynamo system, its circuit simulation and chaotic synchronization via quasi-sliding mode control

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Article Info

ABSTRACT

Rikitake dynamo system (1958) is a famous two-disk dynamo model that is capable of executing nonlinear chaotic oscillations similar to the chaotic oscillations as revealed by palaeomagnetic study. First, we detail the Rikitake dynamo system, its signal plots and important dynamic properties. Then a circuit design using Multisim is carried out for the Rikitake dynamo system. New synchronous quasi-sliding mode control (QSMC) for Rikitake chaotic system is studied in this paper. Furthermore, the selection on switching surface and the existence of QSMC scheme is also designed in this paper. The efficiency of the QSMC scheme is illustrated with MATLAB plots.

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1. INTRODUCTION

Many advances of chaos theory and chaotic systems have been actively carried out in the last few decades [1]-[3]. Classical examples of 3-D chaotic systems include the Rikitake dynamo system [4], and Liu system [5]. New chaotic systems have been also reported in the literature such as Sambas systems [6]-[9], Chen system [10], and Sprott system [11]. Chaos theory has many applications of nonlinear chaotic oscillators [12]-[15]. Idowu *et al.* [13] discussed the adaptive control and circuit implementation of a new 3-dimensional chaotic system with quadratic, cubic and quartic nonlinearities. Hu *et al.* (2 discussed adaptive control methods for Chua's chaotic system [15].

Chaotic systems have applications in neuron models [16]-[18]. Akaishi *et al.* [16] presented a new theoretical model for the nonlinear clinical and pathological manifestations in multiple sclerosis.

Luo *et al.* [17] presented new results for the adaptive passive control of the FitzHugh-Nagumo chaotic neuron model. Hong [18] used adaptive control to discuss the chaos in neurons and devise synchronization schems for Hindmarsh–Rose neuron model.

Chaotic systems have applications in neural networks [19]-[21]. Akhmet and Fen [19] discussed the generation of cyclic and toroidal chaos by Hopfield neural networks. Wang *et al.* [20] derived new results for the synchronization of fuzzy cellular neural network attractors via adaptive control method. Bao *et al.* [21] derived new results for memristive neural networks with threshold electromagnetic induction.

In order to enable synchronisation of chaotic systems for encryption schemes and secure communication devices, many useful control techniques have been devised in the literature such as adaptive control [22], [23], active control [24], [25], fuzzy control [26], [27], backstepping control [28], [29], and sliding mode control [30], [31]. Sliding mode control (SMC) is a special robust approach for variable structure control (VSC) knowledge. Since SMC technique-based control law is effective and guarantees both the occurrence of sliding motion and controlling of the nonlinear systems [32], [33]. So, in this paper, an improved SMC approach is proposed for the synchronous control of chaotic system with Lyapunov-based SMC strategy. The chattering problem, which appeared in these designed sliding mode control laws, is handled thanks to a proposed quasi sliding mode control (QSMC) technique [34], [35].

In this research work, quasi-sliding mode control (QSMC) approach to design the synchronous control of chaotic system with Lyapunov-based SMC strategy is proposed. In this work, a QSMC based master and slave Rikitake chaotic circuits are proposed to guarantee asymptotic synchronization. QSMC design has advantages of fast convergence, robustness and unperturbed by system uncertainties. MATLAB plots are exhibited to illustrate the efficiency of the proposed QSMC control scheme.

2. RIKITAKE CHAOTIC DYNAMO SYSTEM

Explaining research chronological, including research design, research procedure (in the form of algorithms, Pseudocode or other), how to test and data acquisition [1]-[3]. The description of the course of research should be supported references, so the explanation can be accepted scientifically [2], [4]. Rikitake chaotic dynamo system (1958) is described as follows [4]:

$$\begin{cases} \dot{x} = -bx + zy \\ \dot{y} = -by + (z-a)x \\ \dot{z} = 1 - xy \end{cases} \tag{1}$$

We use the notation $X = (x, y, z)$ to represent the state of the Rikitake system (1). Here, a, b are positive parameters.

The Rikitake dynamo system is a famous model that is capable of executing nonlinear chaotic oscillations similar to the chaotic oscillations as revealed by palaeomagnetic study. The behaviour of two disk dynamos coupled to one another was examined by Rikitake [4] in relation to the earth's magnetic field. It was found by Rikitake [4] that reversals of electric current and magnetic field occur in the Rikitake two-disk dynamo system (1) unlike the case of a single disk dynamo.

In this work, we assume that $(a, b) = (3,1)$ and $X(0) = (1,1,1)$. The Lyapunov exponents of the Rikitake system (1) are obtained using MATLAB as:

$$L_1 = 0.1661, L_2 = 0, L_3 = -2.1661 \tag{2}$$

Since $L_1 > 0$ and $L_1 + L_2 + L_3 < 0$, we conclude that the Rkikitake system (1) is chaotic and dissipative. Thus, the system orbits of the Rikitake dynamo system (1) are ultimately confined into a specific limit set of zero volume and the asymptotic motion settles onto a chaotic attractor. The Kaplan-Yorke dimension of the Rikitake dynamo system (1) is found as;

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0767 \tag{3}$$

This shows the high complexity of the Rikitake dynamo system (1). We find that the Rikitake system (1) is invariant under the coordinates transformation,

$$(x, y, z) \mapsto (-x, -y, z) \tag{4}$$

The invariance holds for all values of the parameters a and b. This shows that the Rikitake dynamo system (1) has rotation symmetry about z-axis. The equilibrium points of the Rikitake dynamo system (1) are easily obtained as:

$$E_1 = \begin{bmatrix} 1.8174 \\ 0.5503 \\ 3.3028 \end{bmatrix}, \ E_2 = \begin{bmatrix} -1.8174 \\ -0.5503 \\ 3.3028 \end{bmatrix} \tag{5}$$

A simple calculation shows that E_1 and E_2 are saddle-foci equilibrium points. Hence both are unstable. Thus, the Rikitake dynamo system exhibits self-excited chaotic oscillations. The phase portraits of the Rikitake dynamo system (1) are showed in Figure 1. From this figure, we see that the Rikitake dynamo system (1) exhibits a double-scroll chaotic attractor.
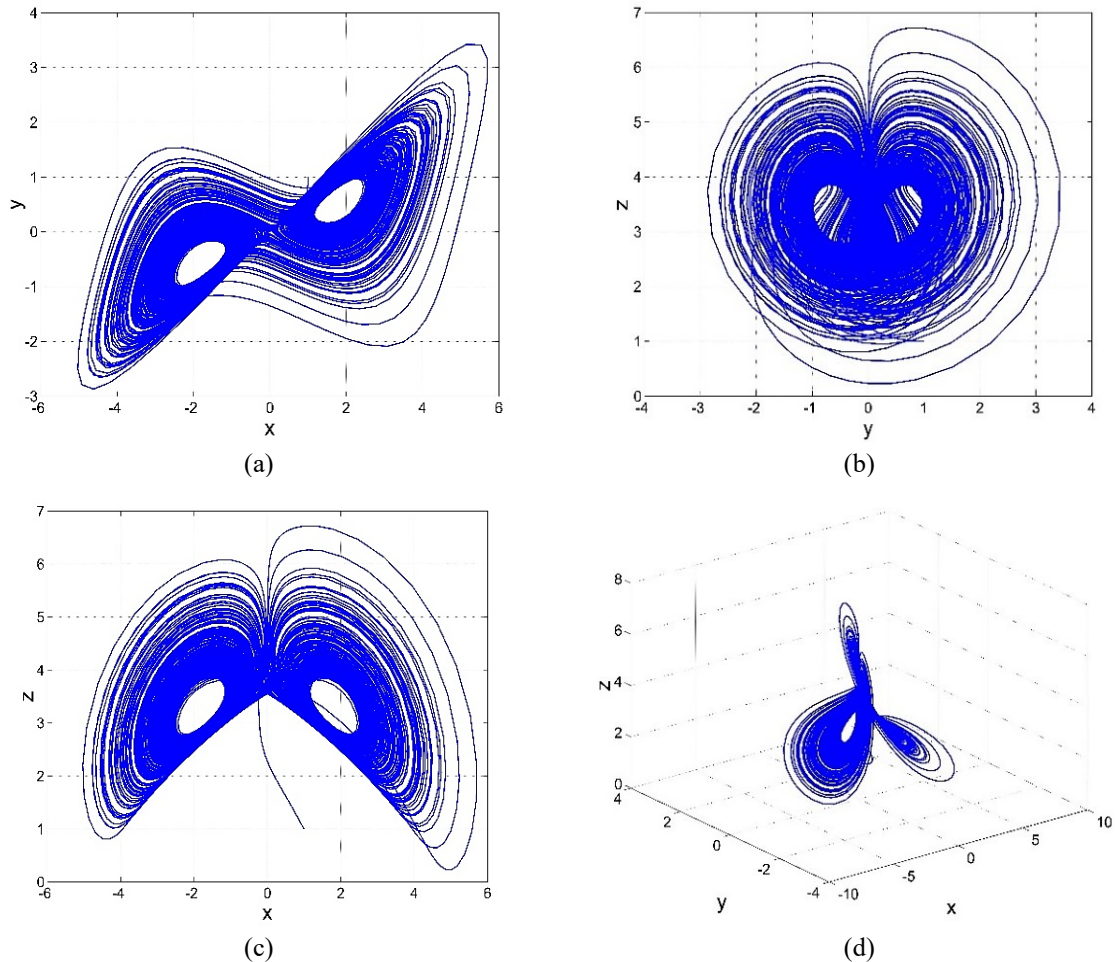

Figure 1. Numerical simulations of the Rikitake dynamo system (1) for $(x(0), y(0), z(0)) = (1,1,1)$ and $(a, b) = (3,1)$: (a) x - y plane, (b) y - z plane, (c) x - z plane, and (d) R^3

3. CIRCUIT IMPLEMENTATION OF THE RIKITAKE DYNAMO SYSTEM

Circuit design of chaotic systems has important applications in engineering [36], [37]. Zhang and Liao [38] discussed chaos synchronization and chaos in coupled memristor-based FitzHugh-Nagumo circuits with memristor synapse and they gave a design of the memristor-based circuit of FitzHugh-Nagumo model. The Rikitake dynamo system (1) is designed in Multisim by making use of the electronic components such as capacitors, resistors, operational amplifiers TL082CD and analog multipliers AD633JN.

For the Multisim circuit design, operating voltage of the operational amplifier is $\pm 15V$. The x, y, z signals of the Rikitake dynamo system (1) are emulated by the output voltages of the integrators. For the circuit implementation of the Rikitake system (1), we adopt the following scaling of the signals x, y, z of the Rikitake system (1):

$$\begin{cases} X = 2x \\ Y = 2y \\ Z = 2z \end{cases} \tag{6}$$

The rescaled Rikitake dynamo system is given by;

$$\begin{cases} \dot{X} = -bX + \frac{1}{2}YZ \\ \dot{Y} = -bY + \left(\frac{1}{2}Z - a\right)X \\ \dot{Z} = 2 - \frac{1}{2}XY \end{cases} \tag{7}$$

We apply the Kirchchoff's laws to derive the circuit presented in Figure 2 as follows;

$$\begin{cases} \dot{X} = -\frac{1}{C_1 R_1}X + \frac{1}{C_1 R_2}YZ \\ \dot{Y} = -\frac{1}{C_2 R_3}Y + \frac{1}{C_2 R_4}XZ - \frac{1}{C_2 R_5}X \\ \dot{Z} = \frac{1}{C_3 R_7}V_1 - \frac{1}{C_3 R_6}XY \end{cases} \tag{8}$$

We take values of the system components as follows:
Let $C_1 = C_2 = C_3 = 1nF$.
Let $R_1 = R_3 = 400k\Omega$ and $R_2 = R_4 = R_6 = 800k\Omega$.
Let $R_5 = 133.33\ k\Omega$ and $R_7 = 200\ k\Omega$.
Let $R_8 = R_9 = R_{10} = R_{11} = 100\ k\Omega$.

The values of various capacitors and resistances were assumed using parameter values of the Rikitake dynamo system and practical considerations. We obtain Multisim outputs of the Rikitake circuit which are shown in Figure 2. It is clear that the Multisim outputs in Figure 3 exhibit the double scroll attractor of the system (7). Thus, the system (7) undergoes chaotic behavior. The Multisim outputs of the Rikitake circuit (7) in Figure 3 are consistent with the MATLAB plots given in Figure 1 of the Rikitake dynamo system (1).

Figure 4 illustrates the Fourier spectral analysis plot in Multisim for the chaotic signal X of the Rikitake circuit (7). The frequency range is 5 kHz, maximum peak 750 Hz. It corresponds to a prevailing frequency of the implementing oscillating loop. The power spectra of the produced signals are broadband, typical of chaotic signals. The frequency is quite low. Thus, the new chaotic system can only be used for low frequency applications. Fourier spectral analysis plot of Figure 4 confirms chaotic oscillations in the Rikitake dynamo system (1).
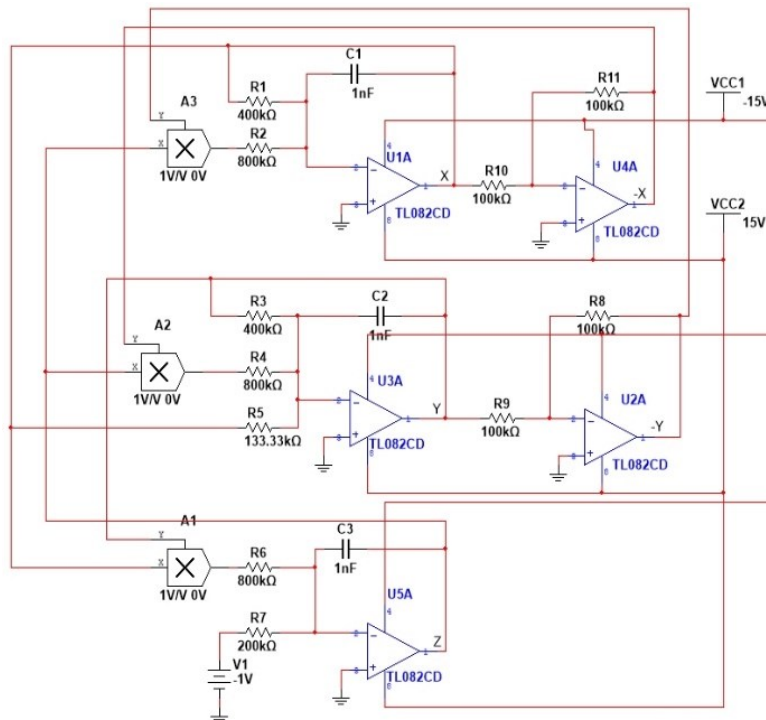

Figure 2. The electronic circuit schematic of the Rikitake dynamo circuit (7)
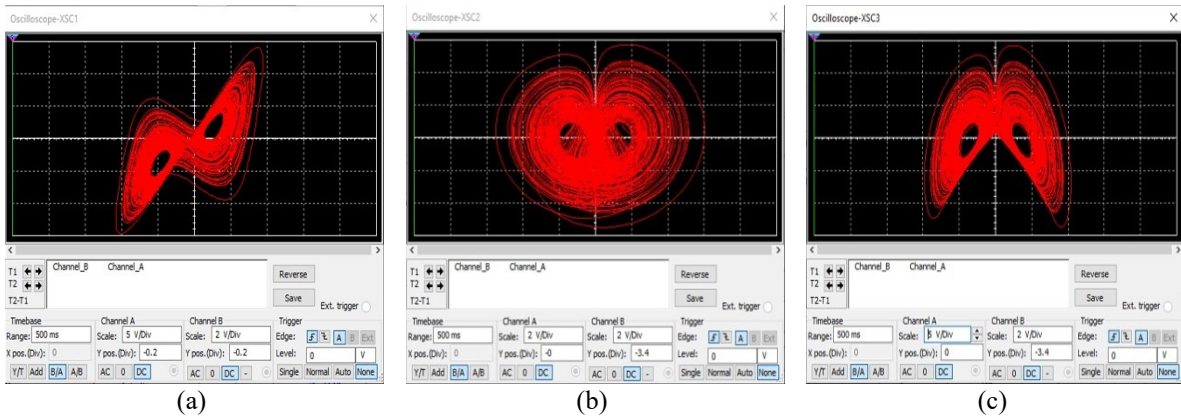
Figure 3. Multisim outputs of the scaled Rikitake dynamo circuit (7) in
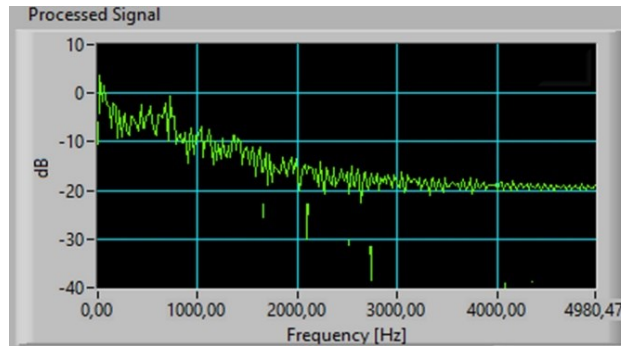(a) x - y plane, (b) $y - z$ plane, (c) $x - z$ plane

Figure 4. Fourier spectral analysis plot in Multisim for the signal X of the Rikitake circuit (7)

4. QUASI-SLIDING MODE CONTROL BASED SYNCHRONISATION OF RIKITAKE CIRCUITS

Sliding mode control (SMC) is proposed to tackle many control problems in engineering applications [39]. There are many variants of SMC such as higher order SMC [39], super-twisting SMC [40], integral SMC [41], robust integral SMC [42], and terminal SMC [43]. In this section, quasi-sliding mode control (QSMC)-based master and slave Rikitake chaotic dynamo systems are proposed to guarantee asymptotic convergence of the synchronization errors of the master-slave Rikitake circuits. The parameters of the Rikitake dynamo systems are taken as in the chaos case, *i.e.* $(a, b) = (3,1)$.

Master system:

$$\begin{cases} \dot{x}_m = -x_m + z_m y_m \\ \dot{y}_m = -y_m + (z_m - 3)x_m \\ \dot{z}_m = 1 - x_m y_m \end{cases} \tag{9}$$

Slave system:

$$\begin{cases} \dot{x}_s = -x_s + z_s y_s + u \\ \dot{y}_s = -y_s + (z_s - 3)x_s \\ \dot{z}_s = 1 - x_s y_s \end{cases} \tag{10}$$

In the slave Rikitake system (10), u is the quasi-integral sliding mode control (QSMC). We next describe the synchronising errors between the Rikitake systems as follows;

$$\begin{cases} e_1(t) = x_s(t) - x_m(t) \\ e_2(t) = y_s(t) - y_m(t) \\ e_3(t) = z_s(t) - z_m(t) \end{cases} \tag{11}$$

The error dynamic system is calculated as follows;

$$\begin{cases} \dot{e}_1(t) = -e_1 + z_s y_s - z_m y_m + u \\ \dot{e}_2(t) = -e_2 + (z_s - 3)x_s - (z_m - 3)x_m \\ \dot{e}_3(t) = -x_s y_s + x_m y_m \end{cases} \qquad (12)$$

The proposed quasi-sliding mode control (QSMC) u can be defined by;

$$u = -w\eta \frac{s}{|s|+\delta} \qquad (13)$$

The switching surface can be chosen as follows:

$$s = e_2 + \lambda e_1 \qquad (14)$$

Here, λ, δ, w are constants and

$$\eta = |[-e_2 + (z_s - 3)x_s - (z_m - 3)x_m] + \lambda[-e_1 + z_s y_s - z_m y_m]| \qquad (15)$$

The quasi-sliding mode control (QSMC) u is designed to derive the error dynamics (11) satisfying the reaching condition $s(t)\dot{s}(t) < 0$ for the sliding surface $s(t) = 0$. The invariance conditions of the sliding manifold are given as follows:

$$s(t) = 0 \text{ and } \dot{s}(t) = 0 \qquad (16)$$

To simplify the notation, we define

$$A = -e_1 + z_s y_s - z_m y_m \text{ and } B = -e_2 + (z_s - 3)x_s - (z_m - 3)x_m \qquad (17)$$

In (15) can be then simplified as follows:

$$\eta = |B + \lambda A| \qquad (18)$$

We also note that;

$$\dot{s} = \dot{e}_2 + \lambda \dot{e}_1 = B + \lambda(A + u) \qquad (19)$$

That is,

$$\dot{s} = B + \lambda A + \lambda u \qquad (20)$$

The proposed Lyapunov function can be provided as;

$$V = \frac{1}{2}s^2 \qquad (21)$$

We calculate the time-derivative of V as follows:

$$\dot{V} = s\dot{s} = s(B + \lambda A + \lambda u) = s(B + \lambda A) + s\lambda u \qquad (22)$$

Thus, it follows that

$$\dot{V} \le \eta|s| - \frac{w\eta s^2}{|s|+\delta} = \eta|s| - w\eta \left(|s| - \frac{|s|\delta}{|s|+\delta}\right) \qquad (23)$$

We note that

$$\frac{|s|\delta}{|s|+\delta} \le \delta \qquad (24)$$

Thus, we can simplify (23) as follows:

$$\dot{V} \le \eta|s| - w\eta(|s| - \delta = (1 - w)\eta \left(|s| - \frac{w\delta}{w-1}\right) \qquad (25)$$

We define

$$\delta_Q = \frac{w\delta}{w-1} \tag{26}$$

With $w > 1$ and $|s(t)| > \delta_Q$, we can conclude that

$$\dot{V} = (1 - w)(|s| - \delta_Q) < 0 \tag{27}$$

By Lyapunov stability theory [44], we conclude that $s(t) \to 0$ asymptotically as $t \to \infty$. By quasi-sliding mode control theory, we have established the chaos synchronization between the master Rikitake system (8) and slave Rikitake system (9).

4.1. Numerical simulations

In this section, we give MATLAB simulation results for the asymptotic synchronization between the master Rikitake system (8) and slave Rikitake system (9). The QSMC control parameters are set as $\lambda = 30$, $\delta = 0.003$ and $w = 1.5$. The initial data for the master system (8) is taken as $x_m(0) = 0.3$, $y_m(0) = 0.2$ and $z_m(0) = 0.1$. The initial data for the slave system (9) is taken as $x_s(0) = 0.6$, $y_s(0) = -0.1$ and $z_s(0) = -0.4$. The synchronization of the respective state trajectories between the master and slave Rikitake dynamo systems (8) and (9) is shown in Figure 5. The synchronization error vector $e(t) = (e_1(t), e_2(t), e_3(t))$ between the master and slave Rikitake circuits is depicted in Figure 6.
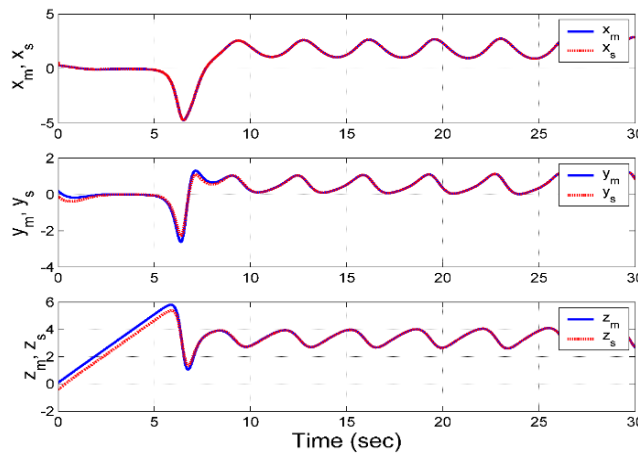

Figure 5. Complete synchronization of the master and slave Rikitake systems (8) and (9)
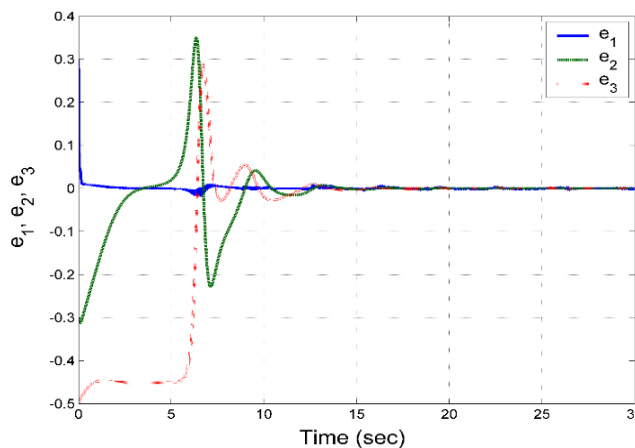

Figure 6. Time-plot of the synchronization error $e(t) = (e_1(t), e_2(t), e_3(t))$ between the master and slave Rikitake dynamo systems (8) and (9)

5. CONCLUSION

In this paper, we gave a brief study of the dynamic properties of the Rikitake circuit (1958) and presented a circuit design for the Rikitake chaotic attractor using Multisim. The main result of this paper is the chaotic synchronisation of Rikitake circuits using a quasi-sliding mode control (QSMC) design. Suitable quasi-sliding mode control controllers were designed by using Lyapunov stability theory. We designed the switching surface for the synchronisation error dynamics. Then a quasi-sliding mode control was considered to guarantee the synchronisation effect on the master and slave Rikitake circuits. MATLAB simulations were shown to illustrate the signal plots of Rikitake circuits and synchronisation results via QSMC. Synchronisation of chaotic systems and circuit design have several applications in engineering areas such as secure communication systems and encryption devices.

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