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Adaptive fuzzy sliding mode control with exponential reaching law and MPL method for the coupled-tank system

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ABSTRACT

This study develops an adaptive fuzzy sliding mode control (ASMC) scheme incorporating an exponential reaching law (ERL) and a minimum parameter learning (MPL) strategy to achieve liquid-level regulation in a coupled-tank system. Such systems are widely used in industrial applications, including chemical and petrochemical processing, water treatment, power generation, and the manufacturing of construction materials, as well as in boilers, evaporators, reactors, and distillation columns. The ERL-based sliding mode controller is formulated to guarantee finite-time tracking of the desired liquid level while effectively suppressing chattering near the sliding surface. The MPL approach is embedded within the fuzzy system (FS), resulting in a single online adaptive parameter, which significantly reduces computational complexity and enhances real-time performance. The stability of the closed-loop system is rigorously established using Lyapunov theory. Simulation studies conducted in MATLAB/Simulink validate the effectiveness of the proposed controller, demonstrating a rise time of 6.1918 s, a settling time of 11.2553 s, zero overshoot, convergence of the steady-state error to zero, and a noticeable reduction in chattering.

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1. INTRODUCTION

Liquid level control is a fundamental problem in industrial process systems, as it directly influences product quality, operational safety, and energy efficiency. This issue commonly arises in chemical and petrochemical industries, water treatment plants, power generation units, and various production processes such as boilers, reactors, evaporators, and distillation columns. Consequently, regulating fluid levels in storage and process tanks has long been regarded as a core challenge in process control engineering [1]–[3].

In recent years, coupled-tank systems have received considerable attention due to their nonlinear behavior and strong inter-tank coupling. Classical control approaches combining proportional–integral–derivative (PID) control and fuzzy logic (FL) have been widely studied and shown to provide satisfactory tracking performance with zero steady-state error under nominal conditions [4]–[6]. Model reference adaptive control (MRAC) schemes have also been reported to outperform conventional proportional–integral (PI) and fuzzy controllers in specific operating scenarios, particularly in terms of robustness and steady-state accuracy [7]. In addition, comparative studies indicate that FL controllers can achieve faster transient responses than traditional proportional (P), PI, and PID controllers for tank-level regulation problems [8].

To further improve performance, adaptive and optimal control strategies have been developed for multi-tank systems. Adaptive fuzzy/proportional–derivative (PD) controllers have demonstrated enhanced tracking capability with reduced steady-state error [9], while optimal control techniques have been employed to mitigate modeling uncertainties and model–plant mismatches, leading to improved transient behavior [10]. Nevertheless, classical PI controllers still suffer from relatively long settling times and high sensitivity to parameter tuning [11].

Driven by these limitations, robust nonlinear control methods have gained increasing interest. Sliding mode control (SMC) is well known for its robustness against uncertainties and external disturbances [12], but conventional SMC is affected by chattering, which degrades tracking accuracy, induces mechanical vibrations, and increases thermal losses in power electronic components [13]. To overcome these issues, this paper proposes an adaptive fuzzy sliding mode control (AFSMC) with an exponential reaching law (ERL) and minimum parameter learning (MPL) for liquid level control of a coupled-tank system, aiming to reduce transient times, steady-state error, chattering effects, and online computational burden.

The remainder of this paper is organized as follows. Section 2 presents the mathematical model of the coupled-tank system. Section 3 describes the design of the proposed AFSMC-ERL-MPL controller. Section 4 discusses the simulation results and performance evaluation, and section 5 concludes the paper.

2. MATHEMATICAL MODEL OF THE COUPLED-TANK SYSTEM

The coupled-tank system comprises two tanks arranged in series, as illustrated in Figure 1 [14], [15]. Let H_1 and H_2 denote the liquid levels in Tank 1 and Tank 2, respectively, while A_1 and A_2 represent their corresponding cross-sectional areas. The interconnecting flow between the two tanks is denoted by Q_{o3}. The variables Q_{i1} and Q_{i2}correspond to the pump inflow rates to Tank 1 and Tank 2, respectively, whereas Q_{o1} and Q_{o2} indicate the outflow rates from each tank.

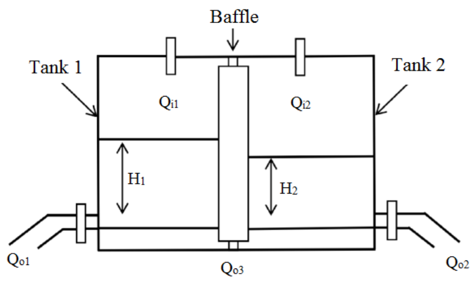

Figure 1. System configuration and structure of the coupled-tank process

Assuming that the inflow rates Q_{i1} and Q_{i2} are fixed, the system reaches steady-state liquid levels H_1 and H_2. Small variations in the inflows, represented by q_1 and q_2, induce corresponding perturbations in the liquid levels, denoted as $h_1$and h_2 [16], [17]. The dynamic behavior of the system is represented by the transfer function in (1) [18]:

$$\frac{h_2(s)}{q_1(s)} = \frac{K_1K_2}{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+(1-K_{12}K_{21})} \tag{1}$$

where,

$\tau_1 = \frac{A_1}{\frac{\alpha_1}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}}$, $\tau_2 = \frac{A_2}{\frac{\alpha_2}{2\sqrt{H_2}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}}$, $K_1 = \frac{1}{\frac{\alpha_1}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}}$, $K_2 = \frac{1}{\frac{\alpha_2}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}}$, $K_{12} = \frac{\frac{\alpha_3}{2\sqrt{H_1-H_2}}}{\frac{\alpha_1}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}}$, $K_{21} = \frac{\frac{\alpha_3}{2\sqrt{H_1-H_2}}}{\frac{\alpha_2}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}}$, $\alpha_1, \alpha_2, \alpha_3$ are proportionality constants.

Within the plant model, the valve (pump actuator) is approximated as a gain element, whose dynamics are characterized by the differential equation presented in (2) [19]:

$$T_c \frac{dq_i(t)}{dt} + q_i(t) = Q_c(t) \tag{2}$$

where, T_c is the time constant of the value/pump actuator, $q_i(t)$ is the time-varying input flow rate, and $Q_c(t)$ is the computed or commanded flow rate.

Define the state variables as (3) and (4):

$$h_2(t) = x_1(t) \tag{3}$$

$$\dot{x}_1(t) = x_2(t) \tag{4}$$

Substituting (3) and (4) into (1), we have (5):

$$\dot{x}_2(t) = -\frac{1-K_{12}K_{21}}{\tau_1\tau_2}x_1(t) - \frac{\tau_1+\tau_2}{\tau_1\tau_2}x_2(t) + \frac{K_1K_2}{\tau_1\tau_2}q_1(t) \tag{5}$$

Define:

$$f(x) = -\frac{1-K_{12}K_{21}}{\tau_1\tau_2}x_1(t) - \frac{\tau_1+\tau_2}{\tau_1\tau_2}x_2(t) \tag{6}$$

The space state of the coupled-tank system as (7) and (8):

$$\begin{cases}\dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(x) + \frac{K_1K_2}{\tau_1\tau_2}u(t) + d(t)\end{cases} \tag{7}$$

$$h_2(t) = x_1(t) \tag{8}$$

where, $x(t) = [x_1(t) \quad x_2(t)]^T$ is the state vector of the system, $q_1(t) = u(t)$ is the control input, $d(t)$ is the disturbance, $|d(t)| \leq D$.

3. DESIGN THE AFSMC-ERL-MPL METHOD

The overall structure of the proposed AFSMC-ERL-MPL control strategy is illustrated in Figure 2. This block diagram shows the closed-loop configuration, in which the reference signal is compared with the measured output to produce the tracking error used by the controller. Based on this error signal, the AFSMC-ERL-MPL algorithm computes the control input to regulate the coupled-tank system dynamics, ensuring accurate liquid-level tracking and robust disturbance rejection. The block diagram of the AFSMC-ERL-MPL method as Figure 2.

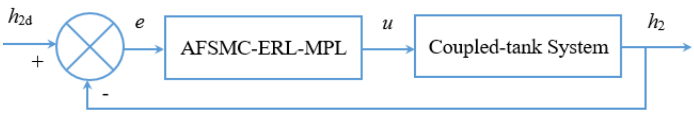

Figure 2. Block diagram of the AFSMC-ERL-MPL

3.1. Design the SMC based on the ERL

The sliding surface is formulated as in (9) [20], [21]:

$$s = ce + \dot{e} \tag{9}$$

where the coefficient c is selected to satisfy the Hurwitz stability condition with $c > 0$. The tracking error is defined in (10) [22] as:

$$e = h_2 - h_{2d} \tag{10}$$

where, h_{2d} denotes the desired liquid level, and h_2 represents the actual liquid level of the coupled-tank system. Taking the derivative of (10), we have (11) and (12):

$$\dot{e} = \dot{h}_2 - \dot{h}_{2d} \tag{11}$$

$$\ddot{e} = \ddot{h}_2 - \ddot{h}_{2d} \tag{12}$$

Taking the derivative of (9), we have (13):

$$\dot{s} = c\dot{e} + \ddot{e} \tag{13}$$

Substituting (7) and (12) into (13), we have (14):

$$\dot{s} = c\dot{e} + f(x) + \frac{K_1K_2}{\tau_1\tau_2}u + d(t) - \ddot{h}_{2d} \tag{14}$$

The SMC based on the ERL is (15) [20], [23]:

$$u = \frac{\tau_1\tau_2}{K_1K_2}\left[-c\dot{e} - f(x) + \ddot{h}_{2d}(t) - \eta sign(s) - \mu s\right] \tag{15}$$

where, $\eta > 0, \mu > 0$.

Now, $\dot{s} = -\eta sign(s) - \mu s + d(t)$, so if we design $\eta \geq D, \mu > 0$, we have (16):

$$s\dot{s} = s\big(-\eta sign(s) - \mu s + d(t)\big) = -\eta|s| - \mu s^2 + sd(t) \leq 0 \tag{16}$$

The component $f(x)$ in (15) is very difficult to measure in actual control. Thus, the fuzzy system (FS) with the MPL method is used in this study to approximate this component [21], [22].

3.2. Uncertainty approximation using an FS

In this section, the unknown nonlinear function $f(x)$ is approximated by an FS $\hat{f}(x)$to implement feedback control. Based on the universal approximation theorem, the fuzzy modeling procedure is developed through the following steps [13].

First, for the input variables $x_1$and x_2, the corresponding fuzzy sets $A_1^{l_1}$ and $A_2^{l_2}$ are defined, where $l_i = 1,2,\ldots,5$. Second, a total of $\prod_{i=1}^{n} p_i = p_1 \times p_2 = 25$ fuzzy rules are constructed to form the FS $\hat{f}(x \mid \theta_f)$, expressed by rules of the form:

$R^{(1)}$: if x_1 is A_1^1 and … and x_2 is A_2^1 then $\hat{f}$ is B^1
⋮
$R^{(25)}$: if x_1 is A_1^5 and … and x_2 is A_2^5 then $\hat{f}$ is B^{25} (17)

where, $l_i = 1,2,\ldots,5,\ i = 1,2,\ p_1 = p_2 = 5$.

Finally, the output of the FS is obtained using standard fuzzy inference mechanisms:

$$\hat{f}\left(x\middle|\theta_f\right) = \frac{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\bar{y}_f^{l_1l_2}\left(\prod_{i=1}^{2}\mu_{A_i^{l_i}}(x_i)\right)}{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\left(\prod_{i=1}^{2}\mu_{A_i^{l_i}}(x_i)\right)} \tag{18}$$

where, $\mu_{A_i^{l_i}}(x_i)$ is the membership function of x_i.

Let $\bar{y}_f^{l_1l_2}$ be a free parameter belonging to the admissible set $\hat{\theta}_f \in R^{(25)}$. A column vector $\xi(x)$ is then introduced, enabling (18) to be rewritten in the compact form given by (19):

$$\hat{f}\left(x\middle|\theta_f\right) = \hat{\theta}_f^T\xi(x) \tag{19}$$

where, $x = [x_1 \quad x_2]^T$, $\xi(x)$ is the $\prod_{i=1}^{n} p_i = p_1 \times p_2 = 25$ the dimensional column vector, and l_1, l_2 denote the corresponding elements [24].

$$\xi_{l_1l_2}(x) = \frac{\prod_{i=1}^{2}\mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{p_1}\sum_{l_2=1}^{p_2}\left(\prod_{i=1}^{2}\mu_{A_i^{l_i}}(x_i)\right)} \tag{20}$$

The membership functions need to be selected according to experience. Moreover, all the states must be known.

3.3. Design of the AFSMC-ERL-MPL

Assume that the optimal parameter vector is defined as in (21) [25]:

$$\theta_f^* = arg \quad \min_{\theta_f \in \Omega_f} \left[sup \left| \hat{f}\left(x| \quad \underset{x \in R^n}{\theta_f} \right) - f(x) \right| \right] \tag{21}$$

where, Ω_f denotes the admissible set of θ_f, i.e., $\theta_f \in \Omega_f$. Accordingly, the nonlinear function f can be expressed as [25]:

$$f = \theta_f^{*T} \xi(x) + \varepsilon \tag{22}$$

where x represents the input vector of the FS, $\xi(x)$ is the fuzzy basis vector, and ε denotes the approximation error, which is bounded by $\varepsilon \le \varepsilon_N$.

The FS is employed to approximate the unknown function f. The input of the FS is selected as $x = [x_1 \ x_2]^T$, and the corresponding FS output is given in (23) [12], [16]:

$$\hat{f}\left(x \middle| \theta_f\right) = \hat{\theta}_f^T \xi(x) \tag{23}$$

Using MPL, define $\phi = \left\| \theta_f^* \right\|^2$, where ϕ is a positive constant, and let $\hat{\phi}$ be an estimation of ϕ. The FSMC-ERL-MPL controller is designed as (24):

$$u = \frac{\tau_1 \tau_2}{K_1 K_2} \left[-\frac{1}{2} s \hat{\phi} \xi^T \xi + \ddot{h}_{2d} - c\dot{e} - \eta sign(s) - \mu s \right] \tag{24}$$

where, $\eta \ge \varepsilon_N + D, \mu > 0$. Then we have (25):

$$\dot{s} = f + \frac{K_1 K_2}{\tau_1 \tau_2} u + d - \ddot{h}_{2d} + c\dot{e} = \hat{\theta}_f^T \xi(x) + \varepsilon - \frac{1}{2} s \hat{\phi} \xi^T \xi - \eta\, sgn(s) - \mu s + d \tag{25}$$

The Lyapunov function is defined as (26) [16], [24]:

$$V = \frac{1}{2} s^2 + \frac{1}{2\gamma} \tilde{\phi}^2 \tag{26}$$

where, $\gamma > 0, \tilde{\phi} = \hat{\phi} - \phi$.
Substituting (25) into the derivative of (26), we have (27):

$$\begin{aligned} \dot{V} &= s\dot{s} + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}} = s \begin{pmatrix} \hat{\theta}_f^T \xi(x) + \varepsilon - \frac{1}{2} s \hat{\phi} \xi^T \xi \\ -\eta\, sgn(s) - \mu s + d \end{pmatrix} + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}} \\ &\le \frac{1}{2} s^2 \phi \xi^T \xi + \frac{1}{2} + \varepsilon s - \frac{1}{2} s^2 \hat{\phi} \xi^T \xi - \eta |s| - \mu s^2 + ds + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}} \\ &= -\frac{1}{2} s^2 \tilde{\phi} \xi^T \xi + \frac{1}{2} + \varepsilon s - \eta |s| - \mu s^2 + ds + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}} \\ &= \tilde{\phi} \left(-\frac{1}{2} s^2 \xi^T \xi + \frac{1}{\gamma} \dot{\hat{\phi}} \right) + \frac{1}{2} - \mu s^2 + \varepsilon s - \eta |s| + ds \le \tilde{\phi} \left(-\frac{1}{2} s^2 \xi^T \xi + \frac{1}{\gamma} \dot{\hat{\phi}} \right) + \frac{1}{2} - \mu s^2 \end{aligned} \tag{27}$$

The adaptive law is designed as (28):

$$\dot{\hat{\phi}} = \frac{\gamma}{2} s^2 \xi^T \xi - \kappa \gamma \hat{\phi} \tag{28}$$

where $\kappa > 0$
Then we have (29):

$$\dot{V} \le -\kappa \tilde{\phi} \hat{\phi} + \frac{1}{2} - \mu s^2 \le -\frac{\kappa}{2} \left(\tilde{\phi}^2 - \phi^2 \right) + \frac{1}{2} - \mu s^2 = -\frac{\kappa}{2} \tilde{\phi}^2 - \mu s^2 + \left(\frac{\kappa}{2} \phi^2 + \frac{1}{2} \right) \tag{29}$$

Define $\kappa = \frac{2\mu}{\gamma}$, then we have (30):

$$\dot{V} \le -\frac{\mu}{\gamma}\tilde{\phi}^2 - \mu s^2 + \left(\frac{\kappa}{2}\phi^2 + \frac{1}{2}\right) = -2\mu\left(\frac{1}{2\gamma}\tilde{\phi}^2 + \frac{1}{2}s^2\right) + \left(\frac{\kappa}{2}\phi^2 + \frac{1}{2}\right) = -2\mu V + Q \quad (30)$$

where $Q = \frac{\kappa}{2}\phi^2 + \frac{1}{2}$.
By applying Lemma 1.3, the solution of the differential inequality $\dot{V} \le -2\mu V + Q$ is obtained as (31):

$$\dot{V} \le \frac{Q}{2\mu} + \left(V(0) - \frac{Q}{2\mu}\right)e^{-2\mu t} \quad (31)$$

Then,

$$\lim_{t\to\infty} V = \frac{Q}{2\mu} = \frac{\frac{\kappa}{2}\phi^2 + \frac{1}{2}}{2\mu} = \frac{\kappa\phi^2 + 1}{4\mu} = \frac{\frac{2\mu}{\gamma}\phi^2 + 1}{4\mu} = \frac{\phi^2}{2\gamma} + \frac{1}{4\mu} \quad (32)$$

From the above-mentioned, we can discern that the overlap precision depends on γ and μ.

4. RESULTS AND DISCUSSION

Figure 3 presents the MATLAB/Simulink schematic of the proposed AFSMC-ERL-MPL control strategy. The parameters of the coupled-tank system are presented as [1]: $A1 = A2 = 32$ (cm^3), $\alpha 1 = \alpha 2 = 14.3$ ($cm^{3/2}/s$), $\alpha 3 = 20$ ($cm^{3/2}/s$), $T_c = 1(s)$, $Q_{imax} = 300$ (cm^3/s), $\tau_1 = 7.445$, $\tau_2 = 6.2$, $K_1 = 0.23267$, $K_2 = 0.1939$, $K_{12} = 0.6453$, and $K_{21} = 0.5378$. $c = 0.35$, $\eta = 12$, $\mu = 2$, $\gamma = 25$, and $d(t) = sin(t)$. These are the parameters of the proposed controller.

Figure 4 depicts the system response and tracking error obtained using the AFSMC-ERL-MPL controller for a reference level of 9 cm. As observed, the actual liquid level accurately follows the desired trajectory, achieving a rise time of 6.1918 s, a settling time of 11.2553 s, zero percent overshoot, and convergence of the steady-state error to zero. These performance indices are summarized in Table 1 and compared with those of the PID FL controller [1] and the conventional PI method [9], both evaluated under identical parameter settings and operating conditions. The comparative results in Table 1 clearly demonstrate the superior control performance of the proposed AFSMC-ERL-MPL approach relative to the reference controllers.

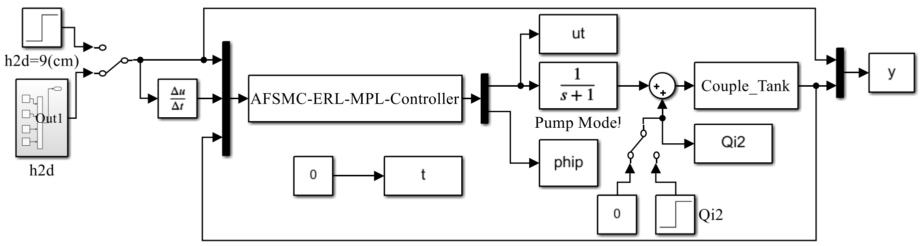

Figure 3. Simulation schematic of the AFSMC-ERL-MPL controller in MATLAB/Simulink

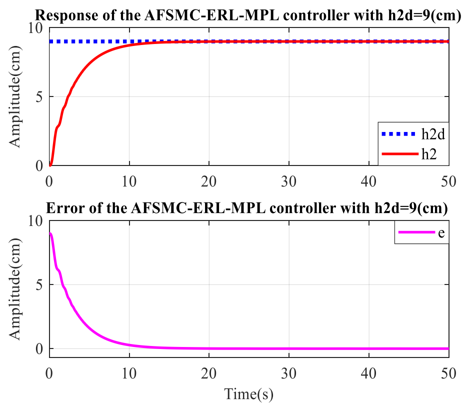

Figure 4. Response and error of the AFSMC-ERL-MPL with h_{2d} = 9 (cm)

Table 1. The achieved quality criteria of the AFSMC-ERL-MPL controller

Quality criteria	AFSMC-ERL-MPL	PID FL [1]	PI controller [9]
Rising time (s)	6.1918	9.95	7.06
Settling time (s)	11.2553	16	32.88
Overshoot (%)	0	0	5.77
Steady state error (cm)	0	0	0

The control signal of the proposed controller is shown in Figure 5. The chattering phenomenon was significantly reduced when using the ERL. This result demonstrates the effectiveness of the AFSMC-ERL-MPL algorithm in controlling the coupled-tank system. Figure 6 presents the estimation result of ϕ.

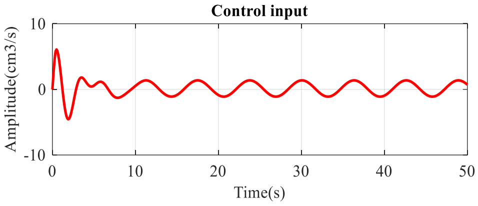

Figure 5. Control input of the proposed controller with $h_{2d} = 9$ (cm)

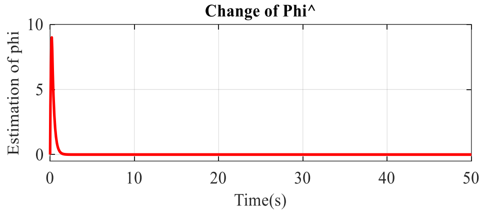

Figure 6. Change of $\hat{\phi}$

Figure 7 illustrates the system response and tracking error of the proposed controller under stepwise varying reference inputs. The liquid level continues to converge to the desired trajectory within a finite time, while the steady-state error tends toward zero. These results confirm the effectiveness and suitability of the proposed control approach for liquid-level regulation in the coupled-tank system. Table 2 presents the quantitative error performance measures derived from the test sample data.

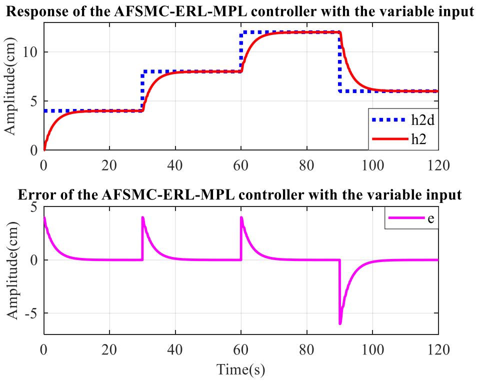

Figure 7. Response and error of the AFSMC-ERL-MPL controller with the variable input

Table 2. Various error performance measures of the AFSMC-ERL-MPL controller

Different measures of error	Average absolute deviation (AAD)	Mean squared error (MSE)	Root mean squared error (RMSE)	Mean percentage error (MPE)	Mean absolute percentage error (MAPE)	Mean relative error (MRE)
h_2	6.9764e-7	9.8655e-10	3.1409e-5	-7.7516e-8	7.7516e-8	7.7516e-6

At $t = 25$ s, the second motor pump connected to the second tank is activated, injecting an additional flow rate of $17\ \text{cm}^3/\text{s}$ into the system. This external disturbance is intentionally introduced to evaluate the robustness of the proposed control strategy. The corresponding output response of the coupled-tank process under this disturbance, when regulated by the AFSMC-ERL-MPL scheme, is illustrated in Figure 8.

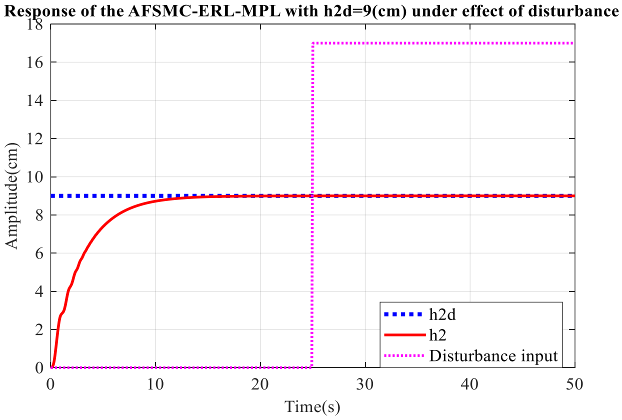

Figure 8. Dynamic response of the coupled-tank system using the AFSMC-ERL-MPL controller in the presence of an external disturbance, with h_{2d} = 9 cm

As shown in Figure 8, the system output rapidly converges back to the desired reference level, demonstrating that the applied disturbance is effectively attenuated and rejected by the proposed controller. Furthermore, Figure 9 presents the controller performance in the presence of white noise, which represents sensor noise with an amplitude of 0.01 W and a sampling period of 0.1 s acting on the system output. Despite the presence of this stochastic perturbation, the controlled system maintains stable operation, continues to track the reference signal accurately, and achieves convergence within a finite time.

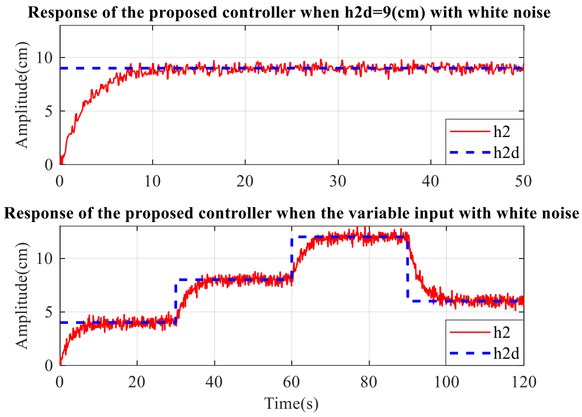

Figure 9. Dynamic responses of the coupled-tank system using the AFSMC-ERL-MPL controller when subjected to white noise disturbances

From the obtained results, the actual liquid level is consistently shown to converge to the reference value within a finite time under all investigated operating scenarios, including nominal conditions, external disturbances, and measurement noise. These results clearly demonstrate the robustness, stability, and effectiveness of the proposed AFSMC-ERL-MPL control strategy in regulating the liquid level of the coupled-tank system.

5. CONCLUSION

This study develops an AFSMC scheme incorporating an ERL and a MPL mechanism for liquid-level regulation in a coupled-tank system. The proposed controller guarantees finite-time convergence of the actual liquid level to the desired reference while effectively suppressing chattering near the sliding surface. MATLAB/Simulink simulation results demonstrate that the controller achieves a rise time of 6.1918 s, a settling time of 11.2553 s, zero overshoot, and vanishing steady-state error, along with a noticeable reduction in chattering effects. Comparative evaluations further confirm that the proposed approach outperforms both the PID FL controller and the conventional PI controller under identical operating conditions. Overall, these results verify the suitability, robustness, and effectiveness of the AFSMC-ERL-MPL strategy for coupled-tank level control applications. Future work will focus on integrating advanced optimization algorithms for parameter tuning and validating the proposed controller through experimental implementation on a real coupled-tank system.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
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Le Minh Thien Huynh		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The research was conducted independently and objectively, without any financial, professional, or personal conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, Le Minh Thien Huynh, upon reasonable request. All simulation models, figures, and analysis scripts were developed by the authors as part of this study and can be shared for academic and non-commercial purposes.

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