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A Novel Approach for Qualitative Imaging of Buried PEC Scatterers

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Abstract

A new linear approach for support reconstruction of impenetrable objects is described and tested in case of scattered field data collected in Ground Penetrating Radar measurement configuration. Starting from the considerations that in high conductivity scatterers the currents induced inside the scatterers are only localized on its boundary and that they take up only few pixels of the entire investigation domain, a sparsity promoting inversion technique is formulated. The flexibility of the approach allows counteracting the specific difficulty to work under aspect limited measurement configurations, as the one at hand. Examples with numerical noisy data are given to demonstrate and validate the effectiveness of the method in localizing and in retrieving the shape of the unknown objects buried in lossy soil.

Keywords*: antennas, inverse source problems, compressive sensing, perfect electric conducting object, qualitative reconstruction, sparsity*

1. Introduction

The reconstruction of qualitative information on an unknown obstacle starting from the measurements of the field it scatters takes on great importance in many testing and diagnostics applications, especially in underground prospecting [1-4].

Solution approaches, which aim at recovering the presence, location and shape of the unknown targets, are usually referred to as *qualitative* methods [5], the most popular being probably the linear sampling method [6,7]. Other inversion techniques, commonly exploited to process Ground Penetrating Radar (GPR) data, are based on Born (BA) or Kirchhoff (KA) approximations. Infact, they manage the problem as a linear one with a very limited range of validity especially in the case of GPR surveys. As a consequence, in practical instances, they cannot provide the electromagnetic properties of the targets but only location and extent of buried targets [1-4]. More specific approaches exist in case of perfect electric conducting (PEC) targets [8-12], also extended to the case of subsurface imaging [13].

In this respect, in this paper an alternative and novel qualitative approach for the reconstruction of the support is described and validate in case of impenetrable objects buried in a lossy soil. The approach is inspired by the fact that, whatever illumination condition, the field inside the target is null and that the scattered field is radiated by some currents just located on the boundary. Inspired by these peculiar features of the induced currents, an *ad hoc* sparsity promoting [14] inversion approach for support reconstruction of conductive obstacles is described and a convenient and reliable 'boundary' indicator is defined and computed, as explained in the following. Note that sparsity promotion and Compressive Sensing theory are very appealing tools for general inverse problem in electromagnetism, as confirmed by the large number of papers published on relevant journals (see f.i. [15-21]), expecially in case of subsurface prospecting [22,23].

The paper is organized as follows. In Section 2, the basic mathematical formulation of the inverse scattering problem in subsurface imaging is recalled. In Sections 3 the proposed approach is introduced and described in detail for PEC buried in lossy soil, while in Section 4 a preliminary assessment of performances is provided considering numerical noisy data. Conclusions follow.

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2. Statement of the Problem

Let us consider the canonical 2D scalar problem (TM polarized fields) with y axis as invariance direction. Let Ω denote the compact, possibly not connected, support of an unknown object with relative permittivity ε_s and electric conductivityσ_s. The non magnetic target is placed at a given depth below the air-soil and is embedded in a soil with features ε_b and σ_b. The unknown buried scatterer is probed with a set of transmitting antennas located in $\boldsymbol{r_t}$ on a curve Γ (along the x axis) placed above the air-soil interface (see Figure 1). The scattered fields are measured by means of receiving antennas located at $\boldsymbol{r_m} \in \Gamma$. Concerning the synthesis of the optimal set-up of transmitting and receiving antennas, different techniques have been developed in literature (see f.i. [24-29]).

By assuming and dropping the time harmonic factor $exp\{j\omega t\}$, the equation which relates the scattered field E_s to the contrast function χ, encoding the electromagnetic properties of the unknown object, can be expressed as:

$$E_s(\boldsymbol{r_m}, \boldsymbol{r_t}) = \int_{\Omega} G_{12}(\boldsymbol{r_m}, \boldsymbol{r'})\, \chi E_t(\boldsymbol{r'}, \boldsymbol{r_t}) d\boldsymbol{r'} = \mathcal{A}_e[W] \tag{1}$$

where E_t is the total field induced inside the investigation domain and $W = \chi E_t$ are the contrast sources, i.e. the currents induced inside the target. G_{12} is the external Green's function for the half space case, i.e., the field radiated in the air by an elementary source placed in the soil [30], while $\mathcal{A}_e$ is a short notation for the integral external radiation operator.

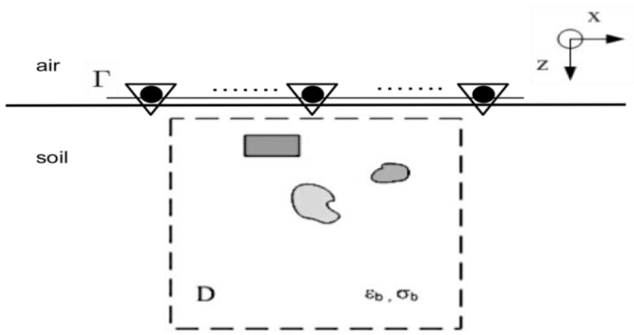

Figure 1. Pictorial view of the multiview-multistatic GPR measurement configuration adopted to collect the scattering experiments. Different transmitting probes (circles) and different receiving ones (triangles) are located on the air-soil interface

The inverse obstacle problem consists in estimating the presence, location and shape of the unknown object, i.e. the support Ω of χ, from the noisy measured scattered field E_s [31]. Unfortunately, the problem is nonlinear and ill posed [32].

In order to deal with such drawbacks, in the following the corresponding inverse source problem, which aims at recovering the currents W from the knowledge of noisy measured scattered field E_s, is considered. This is suggested by the fact that, whatever the performed scattering experiments, the support of the induced currents W is always the same and exactly coincides with the boundary of Ω in case of conductive objects. Note, inverse source problems are still ill posed but at least they are linear.

3. Compressive Sensing Inspired Approach for Qualitative Imaging

Notably, when a generic electromagnetic field is propagating in the space in presence of obstacles, some currents are induced inside the obstacles, which in turn become sources of a new field, known as scattered field.

In case of general targets the induced currents are expected to be different from zero in each point belonging to Ω. However, in case of impenetrable objects, the skin depth is small and the induced currents exist only on the boundary of Ω. As such, these currents take up only few pixels in the investigation domain, so they are sparse in the standard pixel basis representation, i.e. they can be exactly represented by only few non zero pixels.

Starting from these considerations, a possible approach for imaging the shape of a PEC could consist in looking for the sparsest distribution of currents, which are consistent with the measured data. Then, CS theory [14] can be exploited in order to develop an effective and reliable procedure to accurately image the shape and position of conductive targets.

According to CS theory, provided the unknown is sparse in a given basis and the matrix wich relates data and unknown fulfills given properties, it is possible to exactly solve an inverse problem even if the number of independent data is less than the number of total unknown but it is sufficiently larger than the number of coefficients of the representation different from zero [14]. In particular, the sparsest solution for the considered linear problem can be found by minimizing its ℓ_1 norm. Accordingly, by taking ispiration from this theory and by adopting a sparsity promotion procedure, the inverse source problem (1) can be faced as the solution of the following optimization scheme [33]:

$$\begin{aligned} &\min \|\mathrm{W}(\mathrm{r},\mathrm{r_t})\|_1 \\ &\mathrm{s.t.} \quad \|\mathrm{E_s} - \mathcal{A}_\mathrm{e}[\mathrm{W}]\|_2 \leq \delta, \end{aligned} \tag{2}$$

Where $\|\cdot\|_\mathrm{p}$ denotes the ℓ_p-norm and δ is a parameter which depends on the desired accuracy and the amount of noise on the data. However, the sparsity promotion in (2) is not sufficient to univocally solve problem (1) as such a problem is still severely ill posed and many different sparse currents could represent a solution for problem (2).

It is important to underlie that, even if the different incident fields induce different unknown currents$\mathrm{W}(\mathrm{r},\mathrm{r_t})$, independently of the position $\mathrm{r_t}$ of the transmitting antennas, these currents have the same support Ω of the unknown scatter and are always localized on its boundary. In order to enforce this common property between all different experiments, an auxiliary variable Υ can be defined as the upper bound on the amplitude of the electric currents W common to the different scattering illumination conditions. Accordingly, the problem (2) is recast as:

$$\begin{aligned} &\min \|\Upsilon(\mathrm{r})\|_1 \\ &\mathrm{s.t.} \quad \|\mathrm{E_s} - \mathcal{A}_\mathrm{e}[\mathrm{W}]\|_2 \leq \delta, \\ &|\mathrm{W}(\mathrm{r},\mathrm{r_t})| \leq \Upsilon(\mathrm{r}) \ , \quad \forall \mathrm{r_t} \in \Gamma \end{aligned} \tag{3}$$

where the variable Υ depends only on the coordinates of the adopted mesh grid and is expected to be null in all the internal points of the scatterer but for its boundary. For these reasons, in the following Υ is referred to as a ‘boundary’ indicator. It is worth noting that problem (3) belongs to the class of Convex Programming problems.

4. Numerical Examples

In the following, a numerical example dealing with a metallic object is addressed to prove the validity of the proposed approach. The transmitting and receiving antennas are in GPR surface configuration. The scattered field data have been simulated by means of a 2D full wave finite element solver, while the numerical implementation of (3) exploits the CVX Matlab® toolbox [34].

The oval target with dimension of about $0{,}25 \times 0.13\ m^2$, made of alluminum, is buried in a dry soil that exhibits dielectric properties $\varepsilon_b = 4$ and $\sigma_b = 0.1\ mS/m$ (see Figure 1(a)).The imaging domain D, placed just below the air-soil interface, is large $1{,}5 \times 1\ m^2$ and is discretized into $96\ \times\ 64$ cells. The probing array above the air-soil interface is $2{,}5\ m$ long and it is made of

14 evenly spaced antennas. The data matrix is gathered under a multiview-multistatic configuration at a frequency of 400MHz; moreover, it has been corrupted with a random Gaussian noise with a given SNR and processed with no priori information on the targets.

The qualitative reconstruction of the target support, obtained by solving problem (3), is shown in Figure 2(b)-(e). As it can be seen, notwithstanding the aspect limited configuration, the approach allows to localize the metallic targets and to reconstruct its upper contour. Notably, results could be improved by processing multifrequency data. In order to understand the role of δ parameter, in Figure 2 four reconstructions are shown by considering different values. In agreement with [35], some isolated and randomly located pixels exist in the background media when the δ value is too low.

Finally, different values of SNR have been considered to corrupt the scattered data. As it can be seen in Figure 2(f), (g), the method seems to be robust against the noise.

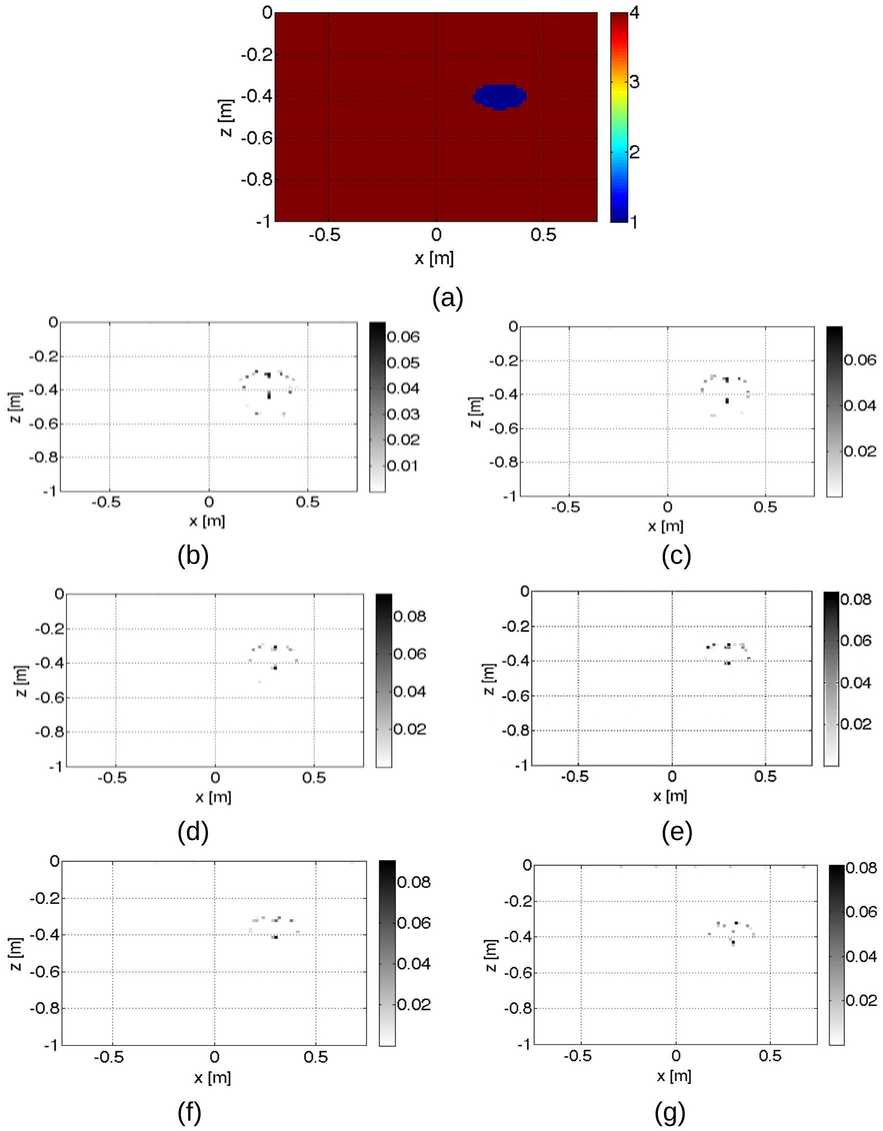

Figure 2. The oval metallic target: (a) Real part of the reference profile. Reconstructed support indicators by considering SNR=30 dB and δ parameter equal to (b) $0.1\|E_s\|_2$, (c) $0.2\|E_s\|_2$, (d) $0.3\|E_s\|_2$ and (e) $0.4\|E_s\|_2$, respectively. Reconstructed support indicators by considering δ parameter equal to $0.4\|E_s\|_2$ and (f) SNR=20 dB and (g) SNR=10 dB, respectively

5. Conclusion

A new method for estimating the shape of metallic targets has been here described and tested in case of GPR prospecting. The key idea is that of solving an auxiliary linear (ill posed) problem in terms of induced currents. Being the measured scattered field radiated by some currents located on the boundary of the support of the target, a properly defined auxiliary function has been defined in each pixel as the upper bound to the different surface currents generating the different scattered fields. Then, the requirement for sparsity in a pixel-based representation of such an auxiliary function has been enforced and a congruity among the different experiments can be considered.

The methods do not require the knowledge of the incident fields but only that of the scattered fields under a sufficiently large number of different scattering experiments. A great advantage can be taken from the possibility to easily extended the approach to a huge variety of different measuring configurations in terms of incident angle and/or also frequency, by only requiring the definition of a single boundary indicator. This flexibility allows counteracting the additional specific difficulty to work under aspect limited measurement configurations.

The approach has been validated with numerical data and multiview-multistatic GPR measurements configuration and an analysis on the role of the noise and the selection of the desired accuracy has been performed.

Future activities will be focused on a deeper understanding of the range of validity of the method in GPR survey and on its extension to the case of dielectric objects buried in lossy soil.

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